Making Mathematics Come Alive With Technology

Dovie Kimmins and Elaine Bouldin
Middle Tennessee State University

Empowering Students Mathematically
Roles of Technology in the Teaching and Learning of Mathematics

- Technology as an Aid in Mathematical Concept and Skill Development
- Technology as an Aid in Mathematical Problem Solving
- Technology as an Aid in Mathematical Reasoning
- Technology as an Aid in Mathematical Communication

Technology and Teacher Preparation

- Emphasis in Mathematics Education
- Technology in School Mathematics - A New Course
- Technology Use in Mathematical Content Courses

EMPOWERING STUDENTS MATHEMATICALLY

A theme which resonates throughout the National Council of Teachers' of Mathematics Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) is that of empowerment. All students should be empowered to actively do meaningful mathematics. Am bitious new goals are proclaimed for all students. All students should value mathematics, have confidence in their ability to do mathematics, become mathematical problem solvers, reason mathematically, and communicate mathematically. Motivating many re commendations in these national standards is the constructivist learning theory - the idea that students actively construct their own knowledge. Accordingly, the role of the instructor changes from that of a dispenser of knowledge to that of a facilitato r of learning. The instructor guides students through experiences in which the students explore, conjecture, verify, generalize, and apply results to other settings and realistic problems - the very activities in which mathematicians engage.

In this context, technology is a valuable tool in the teaching and learning of mathematics, for it has the ability to empower mathematics students as well as mathematics instructors. What better way to empower students than to provide them with their own private investigator, one which will work day and night at the request of the mathematics student who holds power over it, willing to perform tasks repeatedly, however mundane!

ROLES OF TECHNOLOGY IN THE TEACHING AND LEARNING OF MATHEMATICS

Technology can serve at least four roles in the teaching and learning of mathematics. Specifically, technology can aid in (1) mathematical concept and skill development, (2) mathematical problem solving, (3) mathematical

reasoning, and (4) mathematical communication (Kimmins, 1995; Kimmins and Bouldin, 1996).

Technology as an Aid in Mathematical Concept and Skill Development

Specifically in the realm of mathematical concept and skill development, technology (1) empowers students to deal with multiple representations, (2) enhances ability to visualize, (3) increases opportunity to construct mathematical knowledge, and (4) enh ances opportunity for individualized and customized diagnosis, remediation and evaluation (Kimmins, 1995).

With graphing calculators and computer algebra systems, students can explore symbolic, numeric, and graphical representations of functions instead concentrating on the symbolic. Exposing students to multiple representations is important because, in addi tion to the pedagogical advantages of viewing a topic from multiple perspectives, functional models arising from situations encountered in other disciplines often arise in graphical or numeric form instead of symbolic form (NCTM, 1989).

Three - dimensional graphing packages allow students to view solids from different perspectives, thus helping students to see models they had difficulty visualizing otherwise, and at the same time, improving their spatial visualization capabilities (NCTM , 1989; Zimmerman, 1991). An example is the visualization of solids of revolution commonly encountered in first or second semester calculus. An interesting example is that of revolving the graph of + .1 with domain [-1,2] about the x- axis. The resultin g solid of revolution is a "goblet" (Evans & Johnson, 1992). Programs such as Gyrographics from Mathware (formerly from Cipher Systems) allow students to literally revolve a graph about a line and mentally picture the resulting solid of revolution. Then the actual solid of revolution can be drawn and the mental picture confirmed. Finally, the solid can be viewed from any perspective simply by manipulating arrow keys.

Another interesting example in the realm of visualization is the use of Gyrographics to create a graph of a function with a tangent line "riding the curve." Students can literally see that the tangent line is above the graph at points where the graph is concave down and below the graph at points where the graph is concave up. Students literally see the tangent line passing from above to below, or vice-versa, at the point of inflection (Gyrographics Users Manual, 1990).

Tools such as dynamic geometry software allow students to construct mathematical knowledge rather than memorizing theorems dispensed by the instructor. Students can be guided by the instructor to explore, for example, the properties of medians of a trian gle using dynamic geometry software and thus discover that the medians always intersect in a point. In so doing, technology functions as a private investigator gathering data for conjectures (Chazan & Houde, 1989; Education Development Center for Learning Technology, 1991; Klotz, 1991; NCTM, 1989).

One striking set of examples of technology making mathematics come alive in conjunction with concept and skill development is the set of possibilities afforded by the Computer Based Laboratory for Texas Instruments Calculators. Using probes and sensors, physical data can be gathered and stored in the calculator. For example, the motion detector allows a position function, and thus a velocity function, to be graphed for motion detected by the motion sensor. Students can be provided with a calculator-gen erated position or velocity graph and asked to walk in a straight line so as to produce that graph. Instead of an instructor interpreting for students various position and velocity graphs in terms of actual motion, students physically move in such as way as to replicate the graphs (Texas Instruments, 1994).

Finally, a unique example of the power of technology for concept and skill development can be seen in the use of computer and/or calculator programs to simulate sequences of random events. Randomness is the basis for the study of probability and statist ics. Research has shown that many individuals have misconceptions concerning randomness. For example, when asked to write a "typical' sequence of H's and T's resulting from the toss of a coin ten times, most people will write a sequence with no runs of length three, while in fact the probability of getting a run of length three or more is .8 (Moore, 1990). Probability Constructor software by Logal, which simulates various random events and displays the results in various ways is an excellent example of this type of software.

Technology as an Aid in Mathematical Problem Solving

In the realm of mathematical problem solving, technology provides students (1) enhanced ability to focus on the process of problem solving instead of the computational aspect, (2) enhanced ability to solve realistic problems instead of being restricted t o contrived problems having "nice solutions," (3) enhanced opportunity to be introduced to interesting problems and associated mathematical subject matter much earlier than before possible, and (4) increased opportunity to develop mathematical modeling s kills (Kimmins, 1995).

An example of the empowering role of technology as an aid in mathematical problem solving is middle school students solving optimization problems traditionally reserved for calculus students. For example, a typical problem in first-semester calculus tex ts is the following: A box is to be made from a piece of cardboard 10 feet by 2 feet. In order to make the box, a square measuring x feet by x feet is cut from each corner of the cardboard. Find x in order to maximize the volume of the box.

Students can write a calculator or computer program or use the table function on their calculator to generate a table of values of x and the associated volume. The largest volume can be literally observed from the table. Alternatively, the function mo deling the problem can be graphed on a calculator or computer and the maximum point observed. Working with a numeric or graphical representation gives middle school students the power to solve this problem, rather than waiting until calculus provides th em the necessary machinery to use the traditional symbolic approach. The technology empowers the student to concentrate on the process of problem solving rather than on computations (NCTM, 1989).

Technology can afford students increased opportunity to develop mathematical modeling skills. For example, students can be challenged to write a computer or calculator program to simulate the selection of a five-card hand from a 52 card deck. The resul ts of many such simulations would allow students to estimate the probability of various types of card hands before they learned the counting principles necessary to compute the theoretical probabilities. In this exercise students must determine how to mo del a deck of 52 cards with the first 52 counting numbers. In addition to modeling skills, mathematical reasoning and communication skills are developed, as students must use precise language in order to communicate the appropriate tasks to the computer o r calculator.

Technology as an Aid in Mathematical Reasoning In the context of mathematical reasoning, technology has the potential to (1) empower students to gather data in order to form conjectures and apply inductive reasoning and (2) motivate students to think logically in the context of programming a calculat or or computer to perform a desired task. As illustrated previously through the examples of students gathering data concerning medians of a triangle and of students writing calculator or computer code to model a card hand, technology empowers students in the realm of mathematical reasoning (Kimmins, 1995).

Technology as an Aid in Mathematical Communication In the realm of mathematical communication, technology can enhance (1) motivation to communicate mathematics precisely and (2) ability to present mathematical ideas both orally and in writing. Precise language is essential when programming a calculat or or a computer to perform a desired task. Mathematical thought has to be well-formulated and translated into the language of the calculator or computer when, for example, a computer or calculator is programmed to estimate the probability that a quadrat ic equation has real roots under varying conditions on the coefficients. Furthermore, mathematical communication is fostered when students use word processors and presentation software to write and present reports which include mathematical symbols, tabl es, and graphs (Kimmins, 1995).

TECHNOLOGY AND TEACHER PREPARATION

How can universities lead prospective teachers of middle and secondary school mathematics to realize the empowering role of technology in the teaching and learning of mathematics and prepare them to guide the learning process using technology? The Depa rtment of Mathematical Sciences at Middle Tennessee State University has initiated a three-pronged approach: (1) development of a new Emphasis in Mathematics Education at the undergraduate level required of mathematics majors preparing to teach secondary school mathematics, (2) development of a new course, Technology in School Mathematics required of all students in the Mathematics Education Emphasis, and (3) integration of technology into mathematical content courses in which prospective secondary schoo l mathematics teachers are enrolled.

Emphasis in Mathematics Education. As part of revision of the entire undergraduate major, the Mathematical Sciences Department at MTSU has developed an Emphasis in Mathematics Education. All mathematics majors seeking certification must complete this p rofessional program in mathematics education. In agreement with recommendations of the NCTM in the Professional Standards for Teaching Mathematics (1991), the courses and activities composing this new emphasis focus on the nature of mathematics, what ma thematics secondary students should learn and how they should learn it, innovative teaching strategies, alternative assessment, curriculum development, problem solving and the applications of mathematics, use of technology in teaching and learning mathema tics, and state and national trends in mathematics education. The mathematics core consists of 21 hours, specifically Calculus I, II, III; Foundations of Higher Mathematics; Linear Algebra; and Probability and Statistics. Fifteen hours of upper level ma thematics courses consist of College Geometry, Abstract Algebra I, History and Philosophy of Mathematics, and two approved upper-level mathematics electives. Technology in School Mathematics, Teaching Mathematics in Grades 5-8, Teaching Mathematics in Gr ades 9-12, and Topics in Secondary School Mathematics compose the 12 hours requirement in the area of mathematical pedagogy (Kimmins and Bouldin, 1996).

Technology in School Mathematics - A New Course. All students in the Mathematics Education emphasis at MTSU must receive credit in Technology in School Mathematics. Prerequisites of this course are admission to teacher education and completion of the m athematics core, as well as credit in Introduction to Computer Science and Technology in Teaching. Introduction to Computer Science is an introductory course offered by the Department of Computer Science in which students learn a structured programming l anguage. Technology in Teaching, offered by the Department of Educational Leadership, is a part of the secondary education minor. Technology in School Mathematics builds on the skills learned in these prerequisite courses and focuses on mathematics-spec ific technologies and the associated mathematical pedagogy and instructional strategies in subject areas such as pre-algebra, algebra, geometry, calculus, probability, statistics, and discrete mathematics. Computer algebra systems, dynamic geometry softw are, and graphing calculators are emphasized (Kimmins, 1995; Kimmins and Bouldin, 1996).

Technology Use in Mathematical Content Courses. Because "doing mathematics" in the workplace now requires the use of technology in many instances and because of the pedagogical advantages associated with the use of technology in the teaching and learnin g of mathematics, technology is now a component of many of the mathematics content courses in the mathematics major. Thus, prospective teachers of secondary school mathematics experience appropriate uses of technology and see appropriate instructional s trategies using technology modeled in their mathematical content courses. This is vital as we know that "teachers teach as they have been taught."

Mathematics courses which are required of prospective teachers of secondary school mathematics at MTSU and into which technology has been integrated are calculus, abstract algebra, and data analysis. Soon this will be the case for linear algebra and col lege geometry. Furthermore, prospective teachers at MTSU do mathematics using technology when they elect applied statistics, differential equations, or the deterministic and probabilistic models courses. A new course, Seminar in Mathematics with Technol ogy, can also be elected by prospective teachers of secondary school mathematics (Kimmins & Bouldin, 1996).

REFERENCES

Chazan D. & Houde R. (1989). How to use conjecturing and microcomputers to teach geometry. Reston, Va.: National Council of Teachers of Mathematics.

Education Development Center for Learning Technology. (1991). The geometric supposer: triangles. Pleasantville, New York: Sunburst Communications.

Evans B. & Johnson, J. (1992). Classroom demonstrations and activities with the microcomputer. Oklahoma State University. Manual published under a grant from the Oklahoma State Regents for Higher Education.

Gyrographics Users Manual. (1990). Stillwater, Oklahoma.: Cipher Systems.

Kimmins, D. (1995, November). Technology in school mathematics: A course for prospective secondary school mathematics teachers. Paper presented at the Eighth Annual International Conference on Technology in Collegiate Mathematics, Houston, Texas.

Kimmins, D. & Bouldin E. (1996, March). Teaching the prospective teacher: Making mathematics come alive with technology. Paper presented at the Seventh Annual Conference on College Teaching and Learning, Jacksonville, Florida.

Klotz, E. (1991). Visualization in geometry: A case study of a multimedia mathematics education project. In W. Zimmerman & S. Cunningham (Eds.), Visualization in teaching and learning mathematics (pp. 95-104). Washington D.C.: Mathematical Associa tion of America.

Moore, D. Uncertainty. In L. Steen, (Ed.), On the shoulders of giants: New approaches to numeracy. Washington D.C.: National Research Council.

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, Virginia: Author.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school

mathematics. Reston, Virginia: Author.

Texas Instruments. (1994). CBL system experiment workbook. Dallas, Texas: Author.

Zimmerman, W. (1991). Visual thinking in calculus. In W. Zimmerman & S. Cunningham (Eds.), Visualization in teaching and learning mathematics (pp. 127-138). Washington D.C.: Mathematical Association of America.

[Mid-South Instructional Technology Conference Home Page] [Proceedings 1996]